

# Modelling Performance and Emissions from Aircraft for the Aviation Integrated Modelling project

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A new computational method has been developed that simulates the performance of an aircraft and determines the fuel consumption and emissions throughout the flight trajectory by linking the main aircraft aerodynamic characteristics with a model of engine performance. The Performance Emission Simulation of aircraft Operations (PESO) model responds to the needs of the Aviation Integrated Modelling (AIM) project by delivering a computationally fast and reliable model able to simulate aircraft performance, fuel use and emissions. The method is novel in that the airframe aerodynamic characteristics and the performance of the engine are modelled by generic non-dimensional relationships. These non-dimensional characteristics are sufficient to enable accurate determination of the forces acting on the aircraft, the fuel burn of the engine and the key parameters that determine the emissions of pollutants such as nitrous oxides. Within this paper, this new non-dimensional approach is demonstrated and validated using comparisons with flight data from commercial aircraft operations. The results show that the methodology used is sound and that the model can accurately simulate aircraft performance for a range of flight conditions and operating procedures. In future work with the AIM project the method will be applied to investigate novel aircraft technologies, new operating procedures and alternative fuels.

## Nomenclature

$a_{long}$	Component of acceleration in the direction of flight	$TR$	Temperature Ratio
$a_{nor}$	Component of acceleration perpendicular to the flight path	$W$	Weight
$C_L$	Lift coefficient	Greek symbols	
$C_D$	Drag coefficient	$\alpha$	Angle of attack
$c_p$	Specific heat at constant pressure	$\beta$	Angle of climb/descent
$D$	Drag	$\gamma$	Ratio of specific heats
$EI$	Emission Index	$\eta$	Efficiency
$FL$	Flight Level	$\pi$	Generic non-dimensional parameter
$F_N$	Net thrust	Subscripts	
$kt$	Knots	0	Stagnation quantity
$l$	Engine characteristic length	2	Engine inlet
$L$	Lift	3	Combustor inlet
$LCV$	Lower Calorific Value	a	Air
$M$	Mach number	CR	Cruise
$\dot{m}$	Mass flow	f	Fuel
$Mass$	Mass of the aircraft	is	Isentropic
$N_j$	Rotational Speed of LP shaft	p	Generic pollutant
$OPR$	Overall Pressure Ratio	poly	Polytropic
$P$	Pressure	TOC	Top Of Climb
$R$	Air gas constant	Superscripts	
$s$	distance	-	Non-Dimensional
$T$	Temperature		

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## I. Introduction

The Aviation Integrated Modelling (AIM) project has the goal of developing a policy assessment tool for aviation, environment and economic interactions at local and global levels, now and into the future ([www.AIMproject.aero](http://www.AIMproject.aero)). It contains a set of inter-linked modules incorporating the key elements relevant to this goal (see figure 1). These include models for aircraft and engine technologies, air transport demand, airport activity and airspace operations, all coupled to global climate, local environment and economic impact blocks. This architecture of interacting modules is designed to capture major interdependencies and to enable environmental and economic trade-offs to be examined. Further details of all the modules are given in [1].

Embedded within the AIM architecture, the Aircraft Technology Module aims to model the aircraft performance and emissions during all flight phases of any “gate-to-gate” flight operation of a commercial aircraft. The aircraft types to be modelled include current passenger airliners operating within the existing fleet as well as future aircraft that apply proposed airframe and engine technologies. The module is intended to accommodate various operational factors such as load factors, flight trajectories and airline strategies, which are modelled by other modules within AIM. To enable rapid iteration with the other modules the module must be capable of rapidly simulating multiple aircraft flight trajectories across a network of routes.

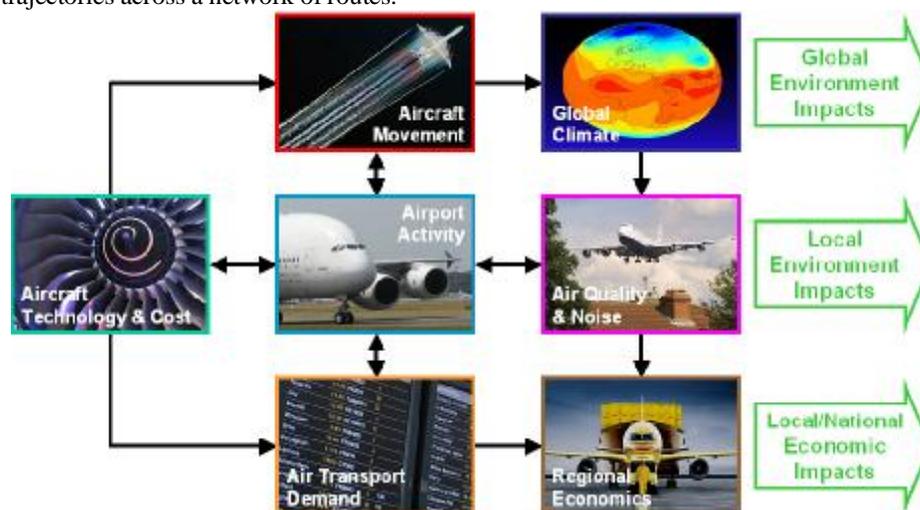


Figure 1: Architecture of the AIM project [1]

There are several existing aircraft trajectory models. Two of the models most widely used by industry to simulate flight operations are the Eurocontrol Base of Aircraft Data (BADA) [2] and the PIANO aircraft analysis tool [3]. The former uses the principle of energy conservation combined with various performance and operating procedure coefficients for a large number of aircraft. These coefficients include those used to calculate thrust, drag and fuel flow and those used to specify standard climb, cruise and descent speed profiles. BADA has been found to be a reliable tool when standard flight procedures are simulated. This is expected given that the required coefficients are derived from conventional airline operational procedures. However, the method is not applicable to non-standard flight operations or novel aircraft technologies. In addition, the absence of a simulation of the thermodynamic cycle of the engine prevents combustion emissions from being calculated. PIANO is a commercial software package that makes use of very detailed aircraft aerodynamic characteristics but uses a relatively inflexible engine model. PIANO applies a dynamic model of the forces on the aircraft to determine the flight characteristics during different flight phases. This approach is found to be more accurate and flexible than that used by BADA, although it is also more demanding computationally.

This paper presents a new computational method for simulating aircraft trajectories that is applied to the Aircraft Technology Module of the AIM project. The method is called PESO (the Performance and Emission Simulation of aircraft Operations) and the details of the methodology are presented below. PESO has been designed to be fast and flexible yet still accurate enough to determine the impact of changes to technology and operational procedures on aircraft emissions. Within this paper, PESO results are compared with recorded data for several flights of the Airbus-A320 aircraft. The comparisons show that PESO accurately reproduces the key features of the measured data for a wide range of flight operations. The study also shows that PESO can reproduce the absolute level of fuel consumption, provided the airframe aerodynamic characteristics are accurately specified.

The PESO method is novel in how it combines a dynamic model of the airframe aerodynamics with a non-dimensional model of the engine. This provides a computationally efficient method that includes sufficient detail to calculate emissions. Using the comparisons with flight recorder data (FDR) this paper makes a useful contribution to the field of aircraft simulation and it provides improved understanding of how aircraft emissions vary with operational factors. This paper will be the basis of future aircraft emissions studies that will apply PESO to more complex scenarios.

## II Overview of the PESO method

PESO simulates the performance of an aircraft and determines the fuel consumption and emissions at any point along the flight trajectory by linking the main aircraft aerodynamic characteristics with a model of engine performance. The method is based on the high speed polar of the aircraft at cruise altitude and on a non-dimensional representation of the engine.

Modelling the flight trajectory of an aircraft involves knowing the relation between the forces acting on the aircraft at each point of the flight mission. PESO considers the aircraft as a point-mass and thus only the forces shown in figure 2 have to be considered.

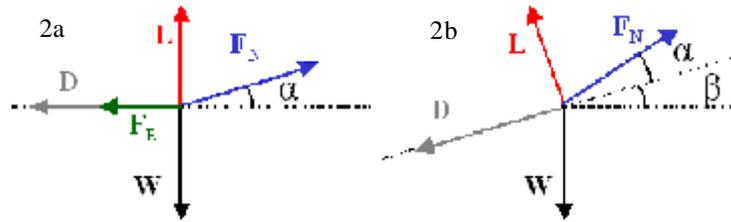


Figure 2: Force balance for an aircraft (a) on the ground (b) airborne

The angle of attack (incidence),  $\alpha$ , is defined as the angle formed by the chord of the wing and the direction of the incoming flow. The angle of climb,  $\beta$ , is defined as the angle formed by the direction of the actual flight path and a plane horizontal to the surface of the earth. From figure 2, the generic equations governing the trajectory of the aircraft can be written as follows:

$$L(\alpha) + F_N \sin \alpha - W \cos \beta = a_{nor} Mass \quad (1)$$

$$-D(\alpha) + F_N \cos \alpha - W \sin \beta = a_{long} Mass \quad (2)$$

For a given mass of the aircraft and assuming that the acceleration in the direction perpendicular to the direction of flight,  $a_{nor}$ , is zero, the system of equations, (1) and (2), contains four unknowns ( $\alpha$ ,  $\beta$ ,  $F_N$ ,  $a_{long}$ ). The dependency of lift and drag on  $\alpha$  is presented in section II.A. In order to find a solution of the two equations, two of the variables have to be fixed. Alternatively, the code can also be used as an optimiser and thus, a combination of the four unknowns is found that meets a given target. Once the net thrust has been calculated, the fuel consumption is determined in order to update the mass of the aircraft for the next point of the trajectory. For a given net thrust, the fuel consumption varies with flight speed and altitude. The modelling of the engine performance for each possible operating condition is presented in section II.B and the procedure to run the model at each phase of the flight is presented in section II.C. Finally, the calculation of the pollutant emissions is presented in section II.D.

### II.A. Airframe Aerodynamics

If the aircraft is represented by a point-mass body, the aerodynamic forces acting on the aircraft can be characterized in a non-dimensional way by  $C_L$  and  $C_D$ . The conventional way of presenting the relationships between these parameters is the  $L/D$  polar as a function of  $C_L$ . To simplify the input data to PESO, only the high speed  $L/D$  polar at a conventional cruise altitude for a range of flight Mach numbers is used. Figure 3 shows an example of such a polar taken from [3]. In the version of PESO considered in this paper, the same high speed polar is used throughout the entire flight mission. For this study, the Reynolds number is considered to be high enough to assume that  $C_L$  and  $C_D$  are independent of Reynolds number.

In order to determine the dependency of the lift coefficient on the angle of attack, the approximation that the two variables are directly proportional has been applied. In order to estimate the constant of proportionality, it has been assumed that at conventional flight Mach number and cruise altitude, the aircraft is flying with optimum  $L/D$  at a typical angle of attack for the aircraft being simulated.

High lift devices are required to create further lift at certain phases of the flight (typically take-off, initial climb, approach and landing). In the current paper, only the ‘clean aircraft’ configuration, *i.e.*, with the high lift devices retracted, will be presented. Some of the limitations of this approximation are considered later in the paper.

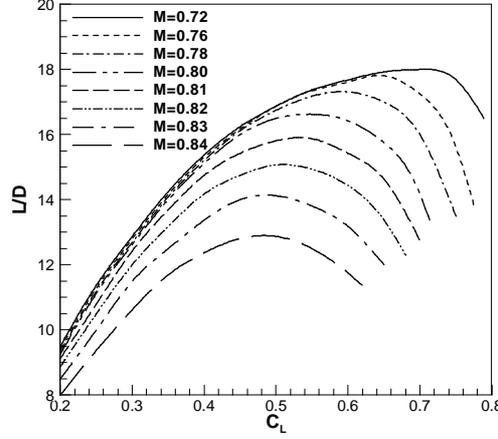


Figure 3: Typical variation of L/D with  $C_L$  and Mach number for a passenger aircraft at conventional cruise altitude, derived from data given in [3]

## II.B. Engine Modelling

In the analysis presented in this paper only turbofan engines are considered. To accurately model the engine at each point of the trajectory, its performance should be simulated for every possible operating condition, *i.e.*, for each altitude, flight Mach number and fuel flow rate to the engine. If each of these possible combinations had to be considered, a large number of calculations would be required. This complexity can be overcome by using a set of non-dimensional engine performance parameters. The full derivation of the appropriate non-dimensional parameters is out of the scope of this paper and can be found in [4] and in [5]. In this paper, only the main assumptions will be mentioned in order to understand the applicability and limitations of the method.

For the analysis presented here it is assumed that the Reynolds number is high and the angle of attack is small so that minor variations do not affect the performance of the engine. Under these assumptions the engine is affected only by the atmospheric conditions, the flight Mach number and the fuel flow to the engines. Of all of these, only the fuel flow can be considered as a control variable. If it is assumed that the final nozzles (core and bypass) are choked then the operating condition of the engine is independent of the static pressure downstream of the nozzle throat (approximately the atmospheric pressure). Hence, any engine performance parameter can be written as a function of the fuel flow and the inlet stagnation pressure and temperature:  $\dot{m}_f$ ,  $P_{02}$  and  $T_{02}$  respectively. Starting from these variables, a non-dimensional parameter can be formed and hence, any generic non-dimensional parameter,  $\bar{p}$ , can be written as

$$\bar{p} = f(\dot{m}_f = \dot{m}_f LCV / (P_{02} l^2 \sqrt{C_p T_{02}})) \quad (3)$$

The conventional non-dimensional parameters for the air mass flow, the rotational speed and temperature and overall pressure ratios are as follows:

$$\bar{m}_a = \frac{\dot{m}_a \sqrt{C_p T_{02}}}{l^2 P_{02}}, \quad \bar{N}_1 = \frac{N_1 l}{\sqrt{gRT_{02}}}, \quad TR = \frac{T_{04}}{T_{02}}, \quad OPR = \frac{P_{03}}{P_{02}} \quad (4)$$

Each of the non-dimensional parameters presented in (3) and (4) can be written as a unique function of any of the other parameters. As shown in [5] the appropriate non-dimensional parameter associated with engine thrust is as follows:

$$\bar{F}_G = \frac{F_G + P_a A_N}{l^2 P_{02}} \quad (5)$$

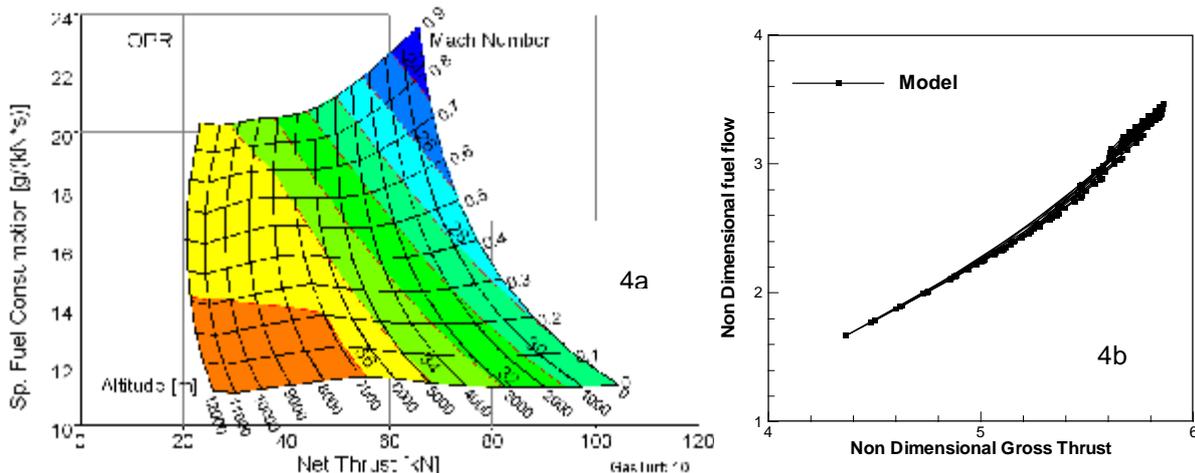
Within PESO the net thrust and the thrust specific fuel consumption (sfc) are required at each point of the flight trajectory. The dimensional specific fuel consumption and net thrust can be calculated by recombining the non-dimensional parameters with the flight condition and the engine size to calculate the dimensional values. Considering  $l^2=A_N$ , the calculations are as follows:

$$\frac{F_N}{P_{02}A_N} = \frac{(F_G - \dot{m}_a V)}{P_{02}A_N} = \left[ \left( \frac{F_G + P_a A_N}{A_N P_{02}} \right) - \frac{P_a}{P_{02}} \right] - M \left( \frac{\dot{m}_a \sqrt{c_p T_{02}}}{A_N P_{02}} \right) \sqrt{(g-1) \frac{T_a}{T_{02}}} = \bar{F}_G - \frac{P_a}{P_{02}} - M \bar{\dot{m}}_a \sqrt{(g-1) \frac{T_a}{T_{02}}} \quad (6)$$

$$sfc = \frac{\dot{m}_f}{F_N} = \frac{\sqrt{c_p T_{02}}}{LCV} \left( \frac{\dot{m}_f LCV}{P_{02} A_N \sqrt{c_p T_{02}}} \right) \bigg/ \left( \frac{F_N}{P_{02} A_N} \right) = \frac{\sqrt{c_p T_{02}}}{LCV} \bar{\dot{m}}_f \bigg/ \left( \frac{F_N}{P_{02} A_N} \right) \quad (7)$$

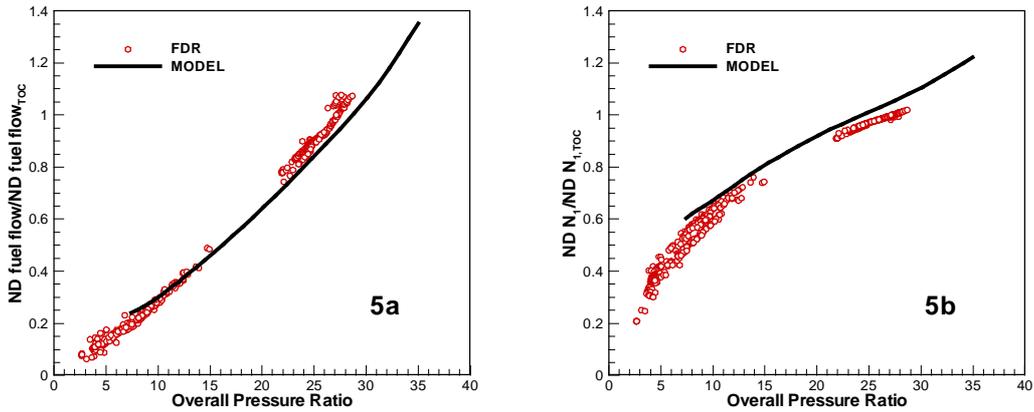
The underlying principle of the method implies that the relationships between the different non-dimensional parameters collapse onto a single working line that is independent of the flight altitude and Mach number. This is true provided the assumption that the final nozzles are choked holds. Using this approach, once the working line is known, if one of the engine non-dimensional parameters can be calculated, then all the others can be found and recombined with the flight condition and engine size to calculate the dimensional values. This requires many fewer calculations to be run. In addition, results can be scaled to any engine that has the same technology level.

To calculate the relationship between the non-dimensional parameters, a commercial gas turbine cycle analysis package called GasTurb is used [6]. Figure 4 shows typical results from GasTurb for a turbofan engine. Figure 4a is a carpet plot showing the variation of specific fuel consumption with the net thrust for different altitudes and Mach numbers. Figures 4a and 4b demonstrate the principle that the non-dimensional fuel flow and thrust are related via a single working line that applies to all flight conditions. The entire carpet plot on figure 4a is replotted with the solid line and symbols labelled as ‘Model’ in figure 4b.



**Figure 4: GASTURB model of an aircraft engine: (a) Variation of sfc with net thrust for different flight conditions (b) Same data plotted in non-dimensional form.**

Figures 5a and 5b show results from the same modelled turbofan engine along with flight recorder data presented in terms of the non-dimensional numbers described above. These figures show that there is a single working line that relates each pair of non-dimensional groups. For the engine model, the working line is calculated by throttling the engine at a given altitude and Mach number. To calculate the non-dimensional numbers from the flight data only raw measurements have been used to calculate the parameters given in equations (3) and (4).



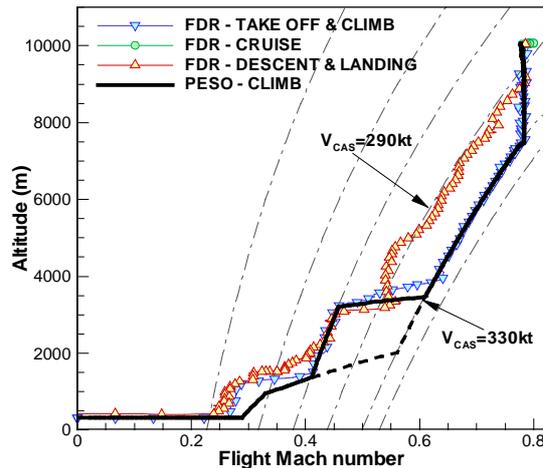
**Figure 5: PESO non-dimensional model of an aircraft engine compared with flight recorder data: Variation of non-dimensional fuel flow (a) and non-dimensional engine speed (b) with OPR**

### II.C. Operations

The PESO model is a flexible tool since any flight operation can be modelled as long as it is within the flight envelope and safety margins. The latter is an important capability for AIM since, for example, the effects of new Air Traffic Management regulations can be studied at local and global scales. Standard flight operation procedures can be modelled by setting at each phase the adequate value of two of the unknowns in equation (1) and (2).

In this paper the take off, initial climb, final descent and landing phases of the flight will not be considered in detail given that for an accurate simulation of these phases, the effect of high lift devices would have to be included. Figure 6 shows a plot of altitude versus Mach number for a specific flight mission (FDR values) and for the corresponding PESO results during climb (cruise and descent have been omitted for the sake of clarity). Lines of constant calibrated air speed (CAS) are also shown in the figure as dash-dotted lines.

For the climbing to top of climb (TOC), a standard operation procedure sets a high thrust setting (85-90% of  $N_1$ ) and a profile of different calibrated air speeds with altitude. In figure 6, the solid line represents the results from PESO when two different calibrated air speeds are entered as a function of the altitude. If the input to the model is to be simplified and a generic profile is to be modelled, only one speed could be fixed, as shown by the dashed line in figure 6. These results demonstrate that fixing calibrated air speeds is an effective way to ensure that PESO reproduces realistic take-off trajectories. The effect of not including high lift devices in the model can be seen in figure 6 where the PESO results cannot reproduce the low speed operation below 1000m. Note that in PESO if the cruise Mach number is reached before TOC, the aircraft keeps ascending at constant Mach number.



**Figure 6: Altitude versus Mach number for a conventional commercial flight during take-off and climb**

The normal cruising procedure involves fixing the Mach number and the angle of climb,  $\beta$ . For constant cruise level the latter will be zero whereas for a stepped cruise, a small angle of climb will be set to change the flight level. For the descent, if a standard flight is to be simulated, the calibrated air speed and angle of descent are fixed.

## II.D. Emissions Modelling

CO<sub>2</sub> has a long-life span and its impact is independent of the location where it is released. More reactive pollutants, like NO<sub>x</sub>, have a local as well as a global effect. The PESO model is able to provide the emissions from aircraft and the specific location where they were emitted. The subsequent impact of these emissions can then be determined by the Air Quality and Noise and the Global Climate modules in the AIM project (see figure 1). In this paper, the only emissions considered are CO<sub>2</sub> and NO<sub>x</sub>. At a point,  $i$ , of the trajectory, the amount of a product of combustion emitted during a time interval  $dt$  is given by

$$P_i = (EI_P \cdot sfc \cdot F_N)_i dt \quad (8)$$

where  $EI_P$  is the emission index of the pollutant. By integrating expression (8) along the flight trajectory, the total amount of a pollutant  $P$  emitted by the aircraft can be calculated. The emission index of a product of combustion depends on factors such as the composition of the fuel, the engine combustor technology, the engine temperatures and pressures and the humidity of the air. For complete combustion of the fuel the emission index of CO<sub>2</sub> depends solely on the mass fraction of carbon in the fuel. However, if the species is a non-stoichiometric product of combustion the emission index can be determined from a severity factor that depends on the operating point of the engine. This severity factor for the NO<sub>x</sub> is given in the following formula given by [7]:

$$EI_{NO_x} = C \times S_{NO_x} \quad \text{where, } S_{NO_x} = \left( \frac{P_{03}}{2965000} \right)^{0.4} + e^{\left( \frac{T_{03}-826}{194} + \frac{6.29-100 \text{ war}}{53.2} \right)} \quad (9)$$

The constant of proportionality,  $C$ , depends on the technology of the combustor and it can be calculated from the ICAO databank [8] as demonstrated by the lines of “best-fit” shown in figure 7.

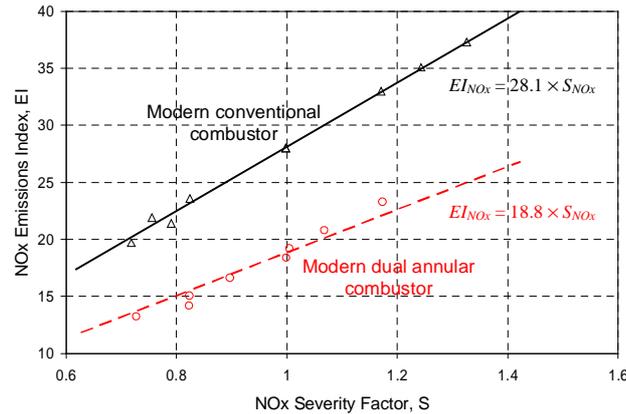


Figure 7: Derivation of the combustor technology constant using the ICAO Database on emissions

## III. Validation

In order to validate PESO, the results from the model have been compared with flight recorder data for an Airbus A320 aircraft. This data includes complete trajectory information as well as selected engine parameters. PESO requires a high speed polar of an A320, which has been derived from [3], and a GasTurb model of the appropriate engine. The engine, a two-shaft unmixed flow turbofan, has been modelled from data available on the CFM web page [9].

The first part of the validation is presented in Section III.A, which compares the performance of the engine model with measurements from real engines. Section III.B then compares aircraft fuel burn data from the FDR with predictions from PESO for flights with a constant level cruise phase. This comparison is carried out for flights at various cruise altitudes, Mach numbers and aircraft weights.

### III.A. Engine Modelling

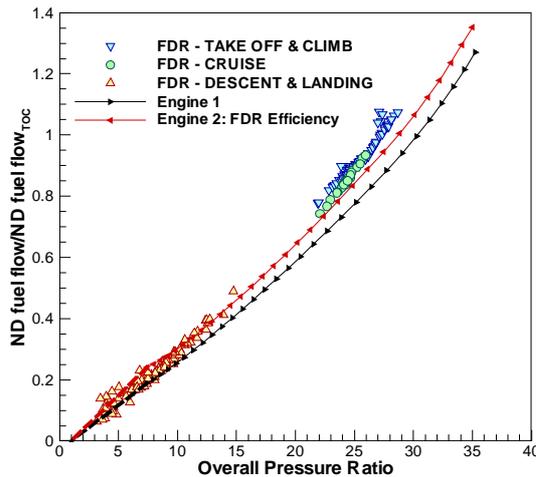
Figure 8 is a plot of non-dimensional mass flow rate versus engine pressure ratio for a complete flight mission. The flight data presented has been divided in three groups according to the phase of the flight: (i) take off and climb, (ii) cruise and (iii) descent and landing. The data corresponding to take off, climb and cruise collapses onto a single line for pressure ratios in the range 20-30. During descent and approach the pressure ratio is lower than 15 owing to

the lower throttle setting during this phase and there is a larger scatter of the data. This is expected due to the unchoking of the final nozzles.

Figure 8 shows two engine working lines generated by GasTurb. The black curve (Engine 1) represents the performance expected for a new modern engine with nominal component efficiencies. The red curve (Engine 2) is a similar model of the engine that has been adjusted to account for in-service performance deterioration. This adjustment has been made by considering the measurements of engine pressure and temperature included in the FDR. The stagnation pressure and stagnation temperature at the exit of the high pressure compressor are included in the flight data and therefore it is possible to determine a polytropic efficiency for the combined low and high pressure compressors using the following equation:

$$h_{poly} = \left( \frac{g-1}{g} \right) \frac{\ln(P_{03}/P_{02})}{\ln(T_{03}/T_{02})} \quad (10)$$

Hence, in the model of the deteriorated engine (Engine 2), the efficiency of the fan and the core compressor are matched to the value from the flight data. Note that in order to carry out this correction, only the flight data corresponding to the cruise phase of the flight is used. During descent, the engine is operating far from the design point and therefore a significant reduction of the polytropic efficiency is expected. To complete the deteriorated engine model, the efficiencies of the high and low pressure turbines were reduced by a similar factor to that applied to the compressors. In addition, the entry temperature to the high pressure turbine was increased to match the pressure ratio and maximum thrust of the original engine (Engine 1). Figure 8 shows that the model of the deteriorated engine (Engine 2) is in closer agreement with the FDR than the original engine model. Only results from PESO using the deteriorated engine model (Engine 2) will be presented from here onwards.



**Figure 8: Variation of non-dimensional fuel flow with OPR for engine flight data and two engine models**

Note that the non-dimensional models of the engines cannot be throttled down to the low overall pressure ratios characteristic of the engine operation during descent. In order to reach these values in the model, the working lines have been artificially extended from a pressure ratio of about 7 to a pressure ratio equal to 1, as shown in figure 8.

### III.B. Constant Level Cruise

Figure 9 shows the fuel consumed during a constant level cruise of a typical A320 flight mission. At the end of cruise, the difference in fuel burn between the FDR and PESO run without any adjustments (labelled PESO on the figure) is about 0.32% of the mass of the aircraft at TOC. For this particular flight this translates into about 200kg of fuel, which is significant. To understand this difference in fuel burn, the values of L/D at which the aircraft is cruising have been investigated. For the PESO model, the value of L/D is set by solving equations (1) and (2) for a constant level cruise (angle of climb equal to zero) and a fixed cruise Mach number. For the flight mission shown in figure 9, the average value of L/D calculated with PESO is about 16.1. For the flight data, the value of the L/D can be estimated from the Breguet-Range equation as follows:

$$\frac{L}{D} \times \frac{1}{sfc} = \frac{-s_{CR} g}{V \ln \left( \frac{W_{end}}{W_{start}} \right)} \quad (12)$$

The right hand side term of equation (12) can be calculated directly from the flight data. The term with the specific fuel consumption on the left hand side can be determined by combining the overall pressure ratio from the flight data with the engine model. Using this approach the average value of L/D for the flight shown in figure 9 was found to be 19.5. If PESO is run using a cruise L/D of 19.5 (labelled PESO + L/D<sub>FDR</sub> in figure 9), the output lies on top of the FDR. This demonstrates that the methodology of the model is sound and that fuel burn data can be accurately reproduced provided the correct lift-to-drag ratio is used.

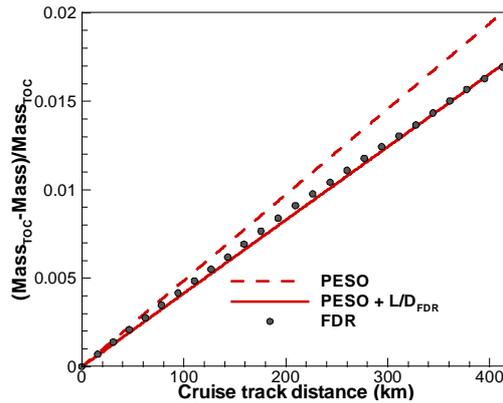


Figure 9: Comparison between FDR and PESO at constant level cruise. Altitude=10060 (FL330), Mach=0.79

The fuel consumed per kilometre flown depends on the altitude, the Mach number, the mass of the aircraft and distance flown. Table 1 shows the effect of the altitude on the fuel consumed per kilometre during a constant altitude cruise. The decrease in the fuel consumed with increasing altitude is captured by PESO, however, the values of lift-to-drag ratio strongly influence the fuel burn.

Mach=0.78; Mass <sub>TOC</sub> /MTOW=0.72; s <sub>CR</sub> =235km					
		Flight Data (FDR)		PESO	
		FL320	FL390	FL320	FL390
Fuel burn kg/km		3.51	2.37	3.04	2.53
L/D		13.0	18.4	14.8	16.8

Table 1: Effect of the cruise altitude on the fuel burn. Comparison between PESO and FDR

A similar result can be seen in Table 2 where the effect of the weight of the aircraft at the beginning of cruise is considered. The same analysis has also been carried out considering the effect of Mach number and length of the cruise with similar conclusions to those above, *i.e.*, the results from PESO show the right trend but the flight data are sensitive to the lift-to-drag ratio. If the flight data average L/D is used as an input to PESO the results from the model are in close agreement with the flight data (not shown in the tables). A consequence of this is that in order to calculate flight trajectories in the context of AIM (figure 1), a distribution of L/D values will have to be introduced to model the range shown in the flight data.

305FL; Mach=0.70; s <sub>CR</sub> =200km					
		Flight Data (FDR)		PESO	
		M <sub>TOC</sub> /MTOW=0.78	M <sub>TOC</sub> /MTOW=0.72	M <sub>TOC</sub> /MTOW=0.78	M <sub>TOC</sub> /MTOW=0.72
Fuel burn kg/km		3.41	2.53	3.21	3.03
L/D		14.5	18.1	16.7	16.0

Table 2: Effect of the aircraft mass on the fuel burn. Comparison between PESO and FDR

#### IV. Applications of PESO

The previous section presented the validation of the methodology for a limited number of constant level cruise phases. This section investigates the capabilities of PESO for a wider range of flight operations. In section IV.A

variations in cruise fuel burn are presented for numerous cruise flight conditions. Section IV.B and IV.C extend PESO to look at aircraft performance during a stepped cruise and a full flight mission.

#### IV.A. Constant Altitude Cruise

Figure 10 shows the fuel consumed per kilometre flown at cruise calculated by PESO for a given cruise length (600km) as a function of Mach number (10a), altitude (10b) and mass of the aircraft at top of climb. In figure 10a, a minimum value of fuel burn is found because for steady level cruise at a given flight level, the ratio of the weight of the aircraft over the Mach number squared determines the value of  $C_L$  and, therefore, the fuel burn. For specific combinations of this ratio and Mach number, the  $C_L$  is optimum. Similarly as the flight level varies so does  $C_L$  leading to an optimum altitude. As expected, the results also show that fuel consumption reduces as aircraft mass is reduced, confirming that non-payload weight needs to be minimised.

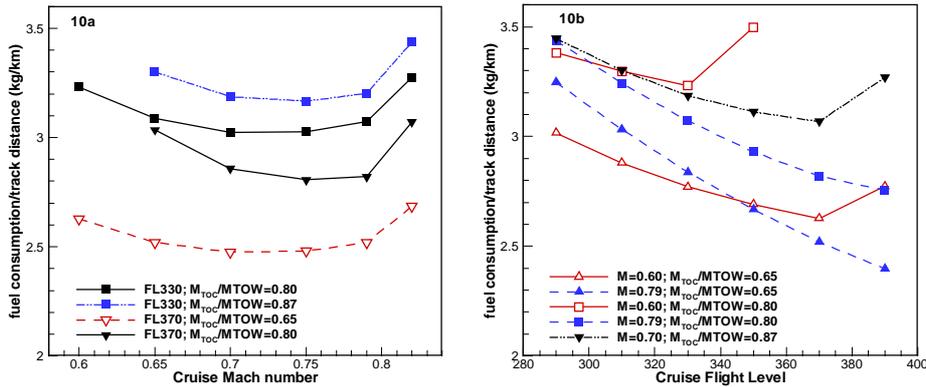


Figure 10: PESO fuel consumption per kilometre as a function of cruise Mach number, flight level and aircraft mass at top-of-climb for a cruise stage length of 600 km

#### IV.B. Stepped Cruise

Standard cruise operations typically involve a stepped climb, especially during long cruise phases. These can be modelled in PESO by specifying the location and altitude of the step (normally multiples of FL10 steps) and using a small angle of climb ( $\sim 1^\circ$ ) up to the target flight level. The Mach number is constant along the entire cruise. Figure 11 shows a comparison between a specific flight and the corresponding result obtained with PESO.

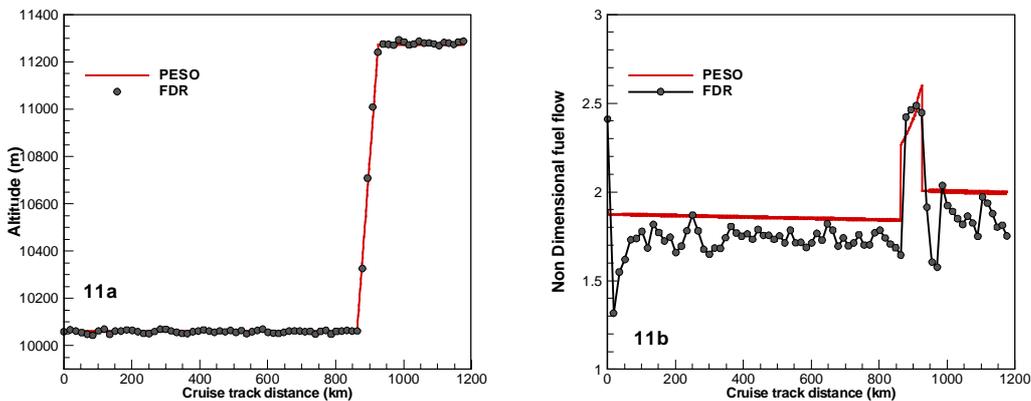


Figure 11: Comparison between FDR and PESO for a stepped cruise. Mach=0.78

The trajectory is shown in figure 11a and the non-dimensional fuel flow during cruise is presented in figure 11b. The non-dimensional fuel flow from PESO shows the same trends as the flight recorder data. These include a decreasing value during the portions of the flight at constant altitude corresponding to the decreasing weight of the aircraft and the sudden increase due to the climb to the next flight level. The magnitude of the step in the non-dimensional fuel flow is captured correctly by the model. The different level between the FDR and the PESO values is due to differences in L/D.

#### IV.C. Full Mission

The performance of PESO along a full flight mission will not be presented in detail in this paper, given that the landing and take off cycle (LTO) is not accurately predicted due to the absence of a model of the high-lift devices. The trends from PESO are compared with flight recorder data for a specific flight profile along the full flight mission, see figure 12. The flight trajectory, figure 12a, shows that PESO captures both the trend and the magnitude of the angle of climb along the climb phase with only the operation of the engine and the calibrated air speed as inputs in equations (1) and (2). Similar trajectories between both sets of data are also seen for the cruise and descent phases. Figure 12b shows the non-dimensional fuel flow and the right trends and levels are seen for the climb and cruise phases although for the descent phase, the results from PESO do not match the flight data as closely as for the previous phases. Figure 12c shows the emission of  $\text{NO}_x$  (from the 2 engines) in kilograms per second for the specific flight under consideration assuming a modern dual annular combustor calculated using the methodology presented in II.D.

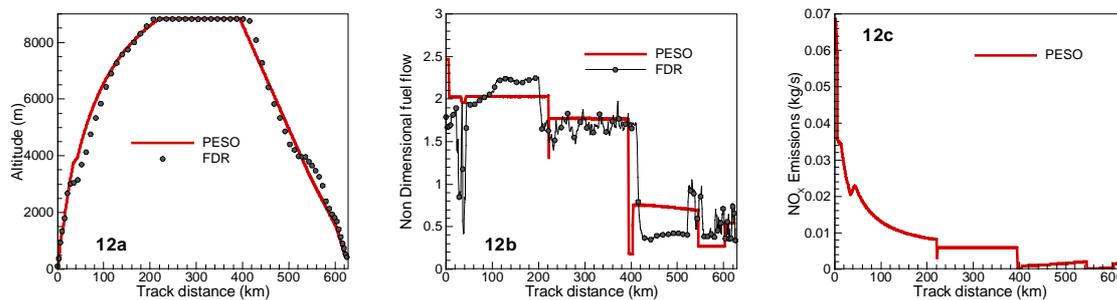


Figure 12: Comparison between FDR and PESO for full flight mission

#### V. Conclusions

In this paper a new approach to the modelling of aircraft trajectories has been presented. The Performance and Emission Simulation of aircraft Operations (PESO) model is based on the use of generic non-dimensional relationships to describe the airframe and the engines, which at each point of the trajectory are linked through the equations of motion. This novel methodology has been validated using flight recorder data. The predictions of the different trends have shown that the methodology is sound although the results are sensitive to the value of aircraft lift-to-drag ratio. The variation in this parameter within the flight recorder data indicates that a distribution of values should be used for the integration of the computational tool within AIM. The PESO model is a flexible tool that has been shown to be capable of reproducing a range of flight operations including stepped cruise phases and complete flights. Aircraft with high lift devices deployed need to be further investigated to improve the simulation of low-speed performance. In addition, now that the methodology has undergone some validation, in future work it could be applied to future aircraft technologies and new operating procedures.

#### Acknowledgements

The authors would like to thank Swiss Air for provision of the flight data presented. They would also like to acknowledge the financial support of the UK Engineering and Physical Sciences Research Council (EPSRC), the Natural Environment Research Council (NERC) and the Sir Arthur Marshall Institute for Aeronautics. Their support is gratefully acknowledged. Particular thanks are also due to past students, Noel Spillane and Hai Le for their work on aircraft emissions modelling. The authors would like to thank Professor Bill Dawes, colleagues from the AIM project and the Institute for Aviation and Environment at the University of Cambridge and at the Massachusetts Institute of Technology for helpful discussions.

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