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**RAPID MODELLING OF AIRPORT DELAY**

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**I. Abstract**

A rapid airport delay model is developed to estimate airport delays with particularly fast runtime, an important requirement for many large-scale air transportation modelling activities. The delay model developed applies a classical steady state queuing theory approach and the cumulative diagram approach to estimate average flight delay in a time varying schedule. The model applies each approach only in periods in which they are applicable, as defined by comparison of period utilisation ratio (average demand over average capacity) to estimated utilisation ratio thresholds. Utilisation ratio thresholds are identified below which the steady state queuing approach is applicable, and above which the cumulative diagram approach is applicable. Between these thresholds, around a utilisation ratio of 1, delay is estimated by linear interpolation between results calculated at each threshold, making the model robust over the full range of utilisation ratios encountered in airport systems. The model results and performance is compared to those of a queuing simulation – a more accurate (but slower) model of airport system delays. Multiple capacity scenarios and demand schedule shapes are analysed over a range of utilisation ratios and bin sizes used to specify the demand and capacity profiles, which is found to be an important parameter. With a bin size of 3 hrs, the delays predicted by the developed model were found to fall within one standard deviation of the simulation results, with maximum differences between the model and simulation below 8 minutes within the range of utilisation ratios likely to be encountered at airports. The developed model run time was found to be in the order of  $10^5$  times faster than the simulation. These results suggest that the developed model may be applied to estimation of average airport delays when errors of 8 minutes are acceptable, and for which a particularly fast runtime is necessary. Such an application is average flight delay estimation at highly congested airports in the Aviation Integrated Modelling project, under development at the University of Cambridge Institute for Aviation and the Environment, where the delay model is run alongside (and integrated with) a number of other model elements to represent the key parts of the global air transportation system.

**II. Key Words**

Airport delay, Rapid runtime, Queuing theory, Classical steady state model, Cumulative diagram approach.

### III. Introduction

Flight delays are a significant problem in many of the world's air transport systems. This is particularly the case in developed systems, such as the United States and Europe. In the United States, average arrival delays at 71 airports were greater than 10 minutes in 2005, with Newark Liberty International Airport experiencing an average arrival delay of 23 min (FAA, 2008). In Europe, average arrival delays at 42 airports were greater than 10 minutes in 2006, with Istanbul International Ataturk Airport and London Luton Airport experiencing average arrival delays of 19 and 18 minutes respectively (EUROCONTROL, 2007).

Flight delays result when system traffic demand approaches or exceeds system capacity. Worldwide demand for air travel has shown significant growth over the past 5 decades – an average growth rate of nearly 9% per year (IPCC, 1999) – and this growth is expected to continue into the future – the UN Intergovernmental Panel on Climate Change (IPCC) forecast a growth rate between 1990 and 2015 of 5% per year (IPCC, 1999). By 2050 conservative estimates predict a 30-110% growth in passenger kilometres travelled over 2005 levels (Berghof et al., 2005), while more aggressive estimates predict an increase of an order of magnitude (Schäfer, 2006). Associated with such growth in demand for air travel is a growth in air traffic to serve that demand. As this traffic demand approaches or exceeds system capacity, delays result.

In developing economies system capacity is expected to grow with increasing air traffic, at least initially, limiting increases in flight delays. However, increases in capacity to match the forecast increases in air traffic are less likely in the industrialised world, where system capacity expansion, particularly at the airport, which is one of the primary bottlenecks in the system (Barnhart et al., 2003), is limited by local community resistance and environmental restrictions. System capacity is thus likely to become an increasing constraint on growth in air traffic, particularly in the industrialised world.

As air traffic demand approaches or exceeds system capacity, and flight delays increase, passengers and airlines are likely to respond. Passengers may increasingly choose alternative modes of transport, or choose not to fly, while airlines may adjust their schedules, routing networks, and types of aircraft they operate in order to reduce delay. Such responses may have a significant impact on the magnitude and distribution of air traffic growth. Forecasting these responses, and their effects, requires integration of models of aircraft performance, passenger demand, airline costs and competition, and flight delay. Such a framework is under development in the Aviation Integrated Modelling Project (AIM) at the University of Cambridge (Reynolds et al., 2007). The modelling of flight delays within such a framework must be particularly computationally efficient as it must be integrated within iteration loops including the other models, and must run a high number of airports globally. However, it does not require the high accuracy of delay models in other applications because of the uncertainties in inputs (e.g. GDP forecasts) and inherent in other models in the framework (e.g. demand model, technology development and turnover models, global climate model, and air quality models). This paper describes an approach for modelling flight delay appropriate to such an application, specifically due to airport capacity constraints. Section IV describes the theory behind the approach developed, followed by a detailed description of the modelling methodology in Section V. Section VI presents results comparing the model to a delay simulation, followed by conclusions in Section VII.

#### IV. Background Theory

Air traffic demand is constrained by airspace and airport capacity constraints, but because of the need to increase the computational efficiency of the delay model being developed, delays due to only airport capacity constraints, which Odoni (2008) suggests are the primary bottleneck in the system, are modelled. Within the airport system a number of constraints exist, including terminal airspace capacity, runway system capacity, taxiway system capacity, gate capacity, and terminal building capacity. Within this system, however, it is the runways that are generally the ultimate capacity constraint (de Neufville and Odoni, 2003, Ch10). Because of the need to increase the computational efficiency of the delay model being developed, delays due to only the runway system capacity are modelled.

As described by de Neufville and Odoni (2003, Ch23), air traffic operations through a airport runway system can be viewed as a queuing system, allowing flow analysis and queuing theory to be used to study and optimise the processes within the system, including estimation of delays. Queuing theory can be applied to the runway system by modelling the entire runway system (including all runways if a multi-runway system is operational) as a server, with the air traffic demand on the system (including both arrivals and departures) modelled as system demand, and the average time in the queue representing average flight delay, as illustrated in Figure 1.

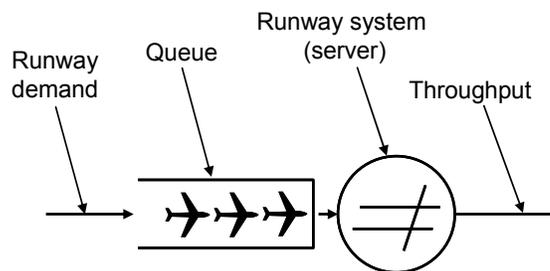


Figure 1. Schematic of the application of a queuing system to an airport system (Idris, 2001).

The most direct approach to modelling a queuing system is through a queuing simulation, in which the demand on the system and the system service rates are represented by probability distributions, from which demand inter-arrival times and server service times are sampled, and any delays resulting when demand enters the system but is not able to be served immediately are recorded. This approach allows the stochastic nature of the arrival and service processes at a runway system to be modelled. The approach can be computationally expensive, especially when the simulation is run repeatedly to calculate average delay results.

Alternatively the queuing system may be modelled by numerically solving a system of equations describing the evolution of the queuing system over time. Kivestu (1976) proposed to model the airport system as an  $M(t)/E_k(t)/k$  system (exponentially distributed (Memoryless) inter-arrival times / service times defined by an Erlang- $k$  distribution /  $k$  servers), and developed a numerical approximation scheme to estimate the state probabilities of the system over time, allowing estimates of average delay and average queue size to be calculated. Malone (1995) further developed the solution method, and proved the approach's appropriateness for application to time-varying queues in airport systems. The approach has good accuracy for estimation of average delays in airport systems, and is less computationally expensive than simulation, but,

depending on the method used to numerically solve the system of equations, may still be too slow for the application proposed in this paper. It has, however, been applied to model airport system delays by a number of authors (e.g. Hebert and Dietz, 1997; Long et al., 1999; Stamatopoulos et al., 2002).

A third alternative to modelling a queuing system is to apply classical steady state queuing theory, for which closed form equations exist for estimation of average delay and queue size when a system has reached steady state. This approach is not typically used to model airport systems, however, because airport queues are typically strongly non-stationary (Barnhart et al., 2003) – demand and service rates often vary significantly during a day, preventing the system from reaching steady state, and often include periods in which demand exceeds capacity for extended periods, which cannot be modelled using this approach. The approach does, however, provide very good computational efficiency because the equations can be solved explicitly. It can only be applied, however, over periods where it is appropriate. One queuing system for which a closed form equation for average delay and queue size exists is a single server with exponentially distributed inter-arrival and service times (the M(t)/M(t)/1 queue – Memoryless arrival rate / Memoryless service rate / single server). This is, however, a less accurate model of the airport system than the M(t)/Ek(t)/k system proposed by Kivestu (1976). For an M(t)/M(t)/1 queue average steady state waiting time in the queue (average delay,  $\bar{D}$ ), and average steady state queue size ( $\bar{L}$ ) are defined as a function of average arrival rate (demand,  $\lambda$ ) and average service rate (capacity,  $\mu$ ) (Larson and Odoni, 1981), as follows:

$$\bar{D} = \frac{\lambda}{\mu(\mu - \lambda)} \quad (1)$$

$$\bar{L} = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad (2)$$

These equations can only be applied when the arrival rate is less than the service rate (demand does not exceed capacity) – i.e. when the system utilisation ratio  $\rho$  (demand over capacity) is less than 1. They can also not be applied when demand approaches capacity closely (i.e.  $\mu - \lambda$  is near zero, or  $\rho$  approaches 1) because average delay tends to infinity. This is because as the utilisation ratio increases, the time required for the system to reach steady state increases. Odoni and Roth (1983) identify a relationship between the time for the system to reach steady state (the characteristic time constant  $\tau$ , or "relaxation time", defined by the time required for the system to settle to within 2% of its steady-state value), the utilisation ratio  $\rho$ , and the average service rate  $\mu$ , as follows:

$$\tau_R = (C_A^2 + C_S^2) / \left( 2.8\mu(1 - \sqrt{\rho})^2 \right) \quad (3)$$

where  $C_A$  and  $C_S$  are the coefficients of variation for the inter-arrival and service times, which are 1 for exponential distributions (as in the M(t)/M(t)/1 system). By rearranging this relation, the maximum value of  $\rho$  for which an M(t)/M(t)/1 system reaches steady state within a specified time  $T$ , ( $\rho_{MM1\max}$ ) can be estimated:

$$\rho_{MM1\max} = \left( 1 - \sqrt{(C_A^2 + C_S^2) / (2.8\mu T)} \right)^2 \quad (4)$$

A fourth alternative to modelling a queuing system is to ignore the stochastic nature of the system, and to apply the cumulative diagram approach – a very simple approach that only

predicts delay if the average system arrival rate (demand,  $\lambda$ ) exceeds the average system service rate (capacity,  $\mu$ ) – i.e. the system utilisation ratio  $\rho$  exceeds 1. When this is the case queue size grows linearly at a rate of the difference between the arrival and service rates, as illustrated in Figure 2. When the arrival rate drops below the service rate the aircraft still in the queue are served first, and the queue size drops linearly at a rate of the difference between the service and arrival rates. The total waiting time in the queue (total delay) is defined by the area under a queue size profile (number of users in the queue with time). The approach is very simple, and therefore has high computational efficiency, but is only accurate for a stochastic system when the average arrival rate greatly exceeds the average service rate (utilisation ratio  $\rho$  is significantly greater than 1). Accuracy may be improved by modelling each flight individually, instead of specifying average demand and capacity profiles in time bins of e.g. 1 hr duration, although this reduces computational efficiency. This approach is applied by Hansen (2002) to compute delay externalities at Los Angeles International Airport.

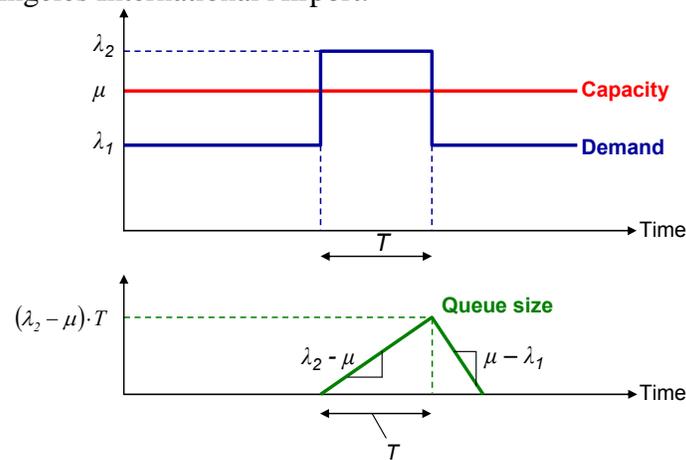


Figure 2. Cumulative diagram approach schematic.

The utilisation ratio above which the cumulative diagram method provides a sufficiently accurate estimate of delay in a stochastic system, given the duration of the time bins in which the demand and capacity profiles are specified, can be identified by comparing the results of the cumulative diagram method with those of another, more accurate, model, such as a queuing simulation. Such a comparison was made, and the values of utilisation ratio  $\rho$  identified at which the cumulative diagram approach estimates delay over a single time bin to within 2% (as used by Odoni and Roth (1983) in the estimation of equation (3)) of the value estimated by an  $M(t)/M(t)/1$  queuing simulation, over a range of average system service rates  $\mu$ , and time bin durations  $T$ . The resulting utilisation ratio threshold, above which the cumulative diagram approach is accurate to within 2% of the simulation result ( $\rho_{CD_{Amin}}$ ), was found to fit equation (5):

$$\rho_{CD_{Amin}} = 1.45 - 0.508 \times 10^{-3} \cdot \mu\tau + 0.338 \times 10^{-6} \cdot (\mu T)^2 \quad (5)$$

The steady state queuing model and cumulative diagram approach are useful below the threshold given by equation (4) and above the threshold given by equation (5) respectively. Because each of their ranges is limited, however, the approaches cannot be applied individually to estimation of airport delay. The ranges are, however, complimentary, with the steady state queuing model applying at low utilisation ratio, and the cumulative diagram approach applying at high utilisation ratio. This paper examines the feasibility of combining the approaches to develop an alternative model that estimates delay robustly over the full range of utilisation ratios

encountered in airport systems. The model developed is described in Section V below, followed by a comparison of its results and performance to a queuing simulation in Section VI.

## V. Modelling Methodology

The basic modelling methodology is to apply the steady state and cumulative diagram approaches described in Section IV to those periods within a time varying schedule for which each approach is appropriate. Thus, in periods for which demand is sufficiently below capacity that an  $M(t)/M(t)/1$  queue reaches steady state (i.e.  $\rho$  is below  $\rho_{MM1max}$ ), equations (1) and (2) are applied. In periods for which demand is sufficiently above capacity that the cumulative diagram approach is accurate (i.e.  $\rho$  is above  $\rho_{CD Amin}$ ), the cumulative diagram approach is applied. This methodology prevents the prediction of unrealistically high delays by the steady state approach as demand approaches capacity ( $\rho$  approaches 1), and the prediction of unrealistically low delays by the cumulative diagram approach when demand only just exceeds capacity ( $\rho$  is above 1, but below  $\rho_{CD Amin}$ ).

In periods in which neither approach is appropriate (i.e. when  $\rho$  is between  $\rho_{MM1max}$  and  $\rho_{CD Amin}$ ) a linear interpolation between results for the two approaches at the respective utilisation ratio thresholds is applied. This methodology is illustrated in Figure 3.

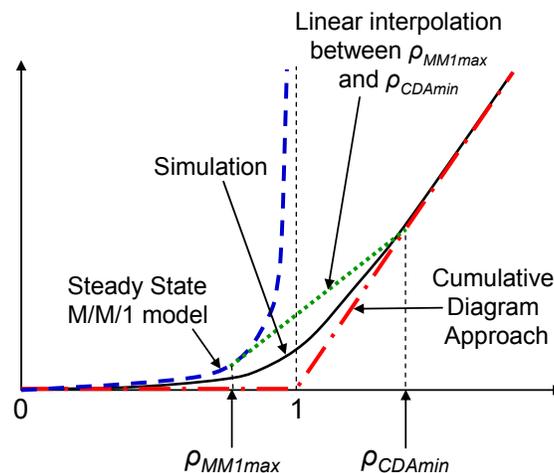


Figure 3. Modelling Methodology.

The methodology provides good accuracy either side of the utilisation ratio thresholds, where the respective models are applicable, but provides limited accuracy (as can be seen in comparison to simulation results in Figure 3) in the transition region, where the linear interpolation is applied.

The approach is trivial to apply to a non-time varying queue, but when applied to a time varying queue, such as an airport schedule, the propagation of the accumulated delay between time bins requires careful implementation. Any clients not served in a time bin remain to be served by the following time bin, where they are served before the clients arriving into the system in that bin.

The approach applied is to calculate the change in queue size in each time bin, chronologically through the day, as a function of the demand and capacity in each respective time bin, and the queue size at the start of the time bin. The queue size at the end of each time bin defines the

queue size at the start of the next time bin. Average delay incurred by the clients is estimated by integrating the queue size profile through all time bins and dividing by the total number of users served.

The queue size profile for each time bin is estimated as follows:

1. If demand is below the maximum demand threshold for the steady state approach defined by  $\rho_{MM1max}$  the queue size at the start of the bin is reduced at the rate of the difference between the bin capacity and demand, until it reaches the steady state queue size for an  $M(t)/M(t)/1$  queue calculate using equation (2). The queue size is then maintained at this value until the end of the bin, and is the starting queue size for the following bin. If the time bin is not long enough for the queue size to reduce to this value, the queue size at the end of the bin is the value to which it has reduced by that time.

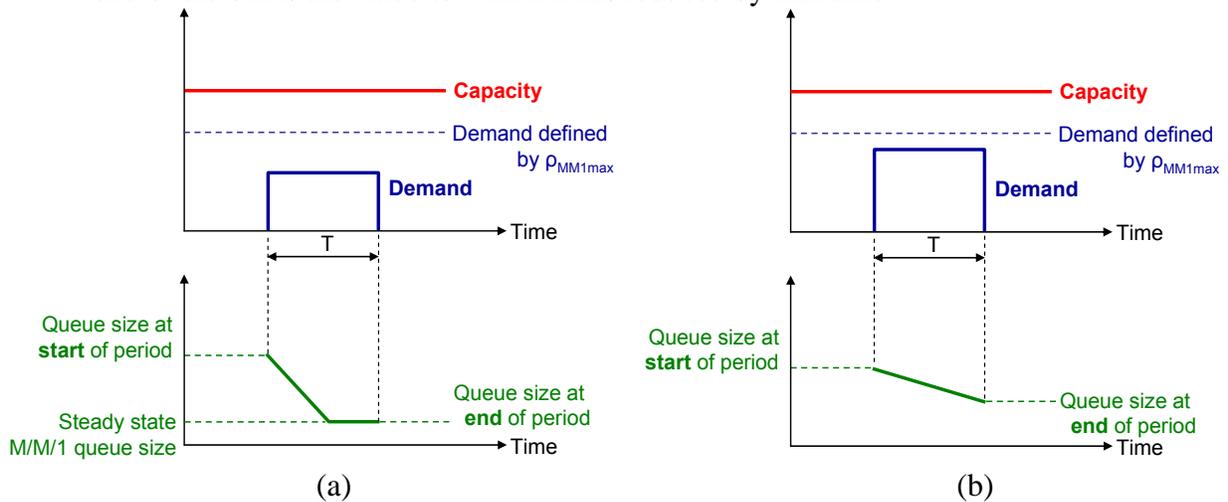


Figure 4. Modelling methodology when  $\rho < \rho_{MM1max}$ , when (a) queue size drops to the steady state value, and (b) when it does not.

2. If demand is above the minimum demand threshold for the cumulative diagram approach defined by  $\rho_{CDAmin}$  the queue size at the start of the bin is increased at the rate of the difference between the bin demand and capacity, until the end of the bin. The queue size at the end of the bin is the value to which it has increased by that time.

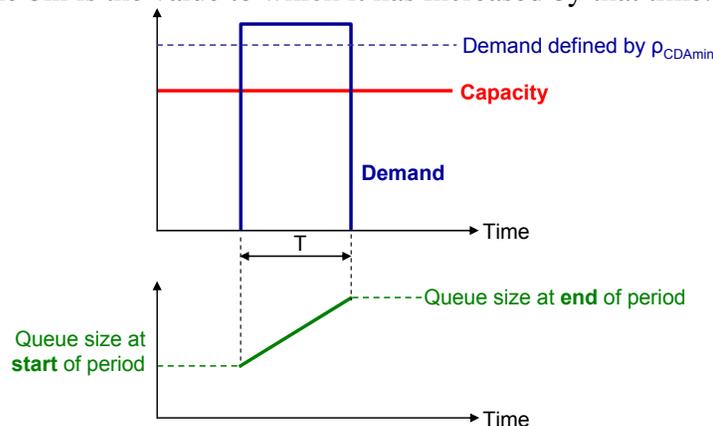
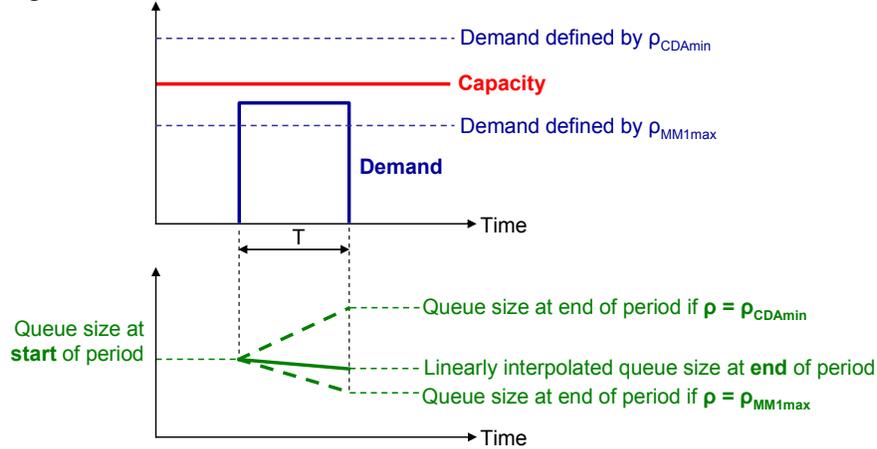


Figure 5. Modelling methodology when  $\rho > \rho_{CDAmin}$ .

3. If demand falls between the maximum demand threshold for the steady state approach defined by  $\rho_{MM1max}$ , and the minimum demand threshold for the cumulative diagram approach defined by  $\rho_{CDAmin}$ , the queue size profile for the bin is calculated by linearly interpolating between the queue size profiles calculated for demand at each threshold. At the maximum demand threshold for the steady state approach the queue size profile is calculated using the approach described in 1 above, while at the minimum demand threshold for the cumulative diagram approach, the queue size profile is calculated using the approach described in 2 above. The actual queue size profile for the bin is then calculated by linearly interpolating between the queue size profiles at the two thresholds according to the ratio of the actual demand in the bin, and the demand thresholds.



**Figure 6.** Modelling methodology when  $\rho$  falls between  $\rho_{MM1max}$  and  $\rho_{CDAmin}$ .

The number of users to be processed by the queue through all time bins is the sum of the demand profile, through all time bins, subtracting the queue size at the end of the final bin. The total delay incurred by the served clients is the integral of the queue size profile, subtracting the delay incurred by the clients not served at the end of the last bin. These clients can be assumed to be cancelled. Average delay is estimated by dividing the total delay incurred by the served clients by the number of users served.

Section VI presents results of the model, applied to time varying queues, and compared to a simulation applied to the same queues.

## VI. Model Comparison to Queuing Simulation

The results and performance of the model developed are described in this section for a series of time varying schedules and system capacities. Two different schedules were analysed – a “banked” schedule based on that for Dallas-Fort Worth International Airport in 2000 (DFW), and a “flat” schedule based on that for San Francisco International Airport in 2000 (SFO) (FAA, 2008). These schedules are presented in Figure 7 below.

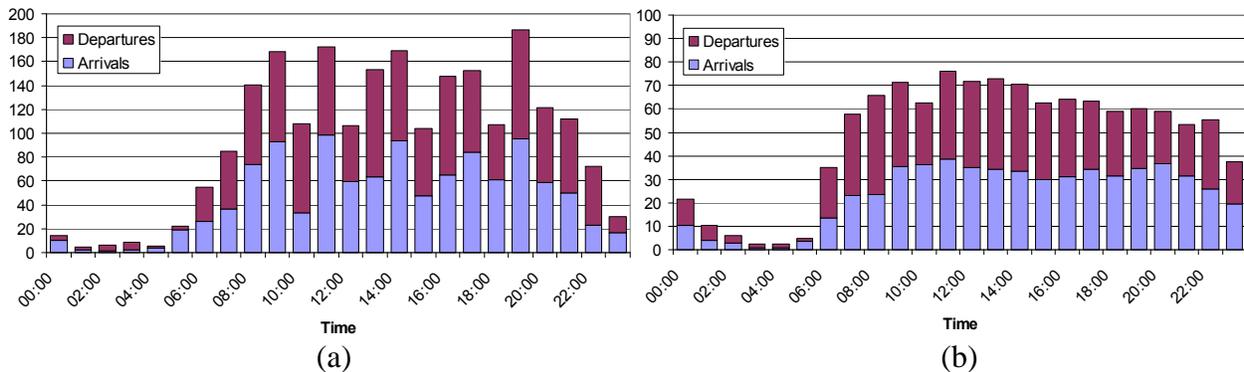


Figure 7. (a) Banked Schedule – DFW, 2000; (b) Flat Schedule – SFO, 2000.

Three average capacity scenarios were examined: 40 aircraft/hr, 100 aircraft/hr and 300 aircraft/hr. These cover the range of airport capacities expected to be encountered by the model, including all those capacities in the FAA Airport Capacity Benchmark Report 2004 (US DOT et al., 2004). Airport capacity was assumed to be constant throughout the day.

Demand, as defined by the two schedules in Figure 7, was scaled to define a range of average utilisation ratios from 0.1 to 1.3, allowing a range of demand and utilisation ratios to be analysed for each capacity scenario. Average utilisation ratio is defined as the ratio of the total demand from 06:00 to 24:00, and the total capacity over the same period. This period was selected because there is very little demand between 24:00 to 6:00 in either of the schedules analysed. Capacity is no lower during this period than during the rest of the day, however, so estimation of the utilisation ratio over the entire day yields un-intuitively low values.

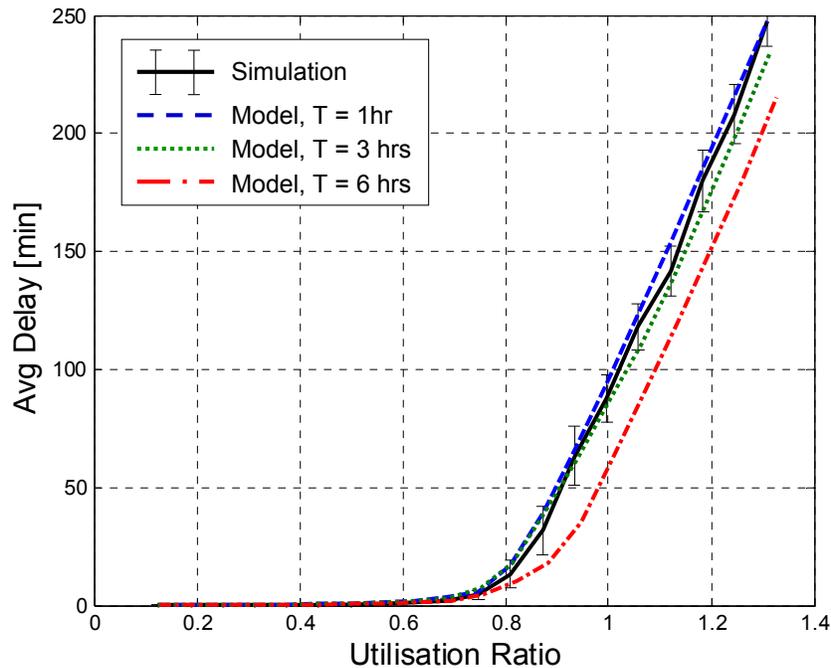
Demand was defined according to the schedules in Figure 7 for a 24 hr period, but this was followed by a second 24 hr period in which demand was zero but capacity maintained at the level of the first period. This allowed any clients that had not been served in the first 24 hr period to be served. This is not realistic, but allows the model and simulation to be compared without consideration of flight cancellation policies.

While the schedule was specified per 1 hour, the model was run with demand specified in different time bin sizes. Small bin sizes (such as per hour) allow the variability of the schedule to be captured in detail, which is important for a highly banked schedule such as that in Figure 7(a), where banks are only separated by an hour. However, small bin sizes require more restrictive utilisation ratio thresholds (according to equations 4 and 5), resulting in application of the linear interpolation between the steady state approach and the cumulative diagram approach over a

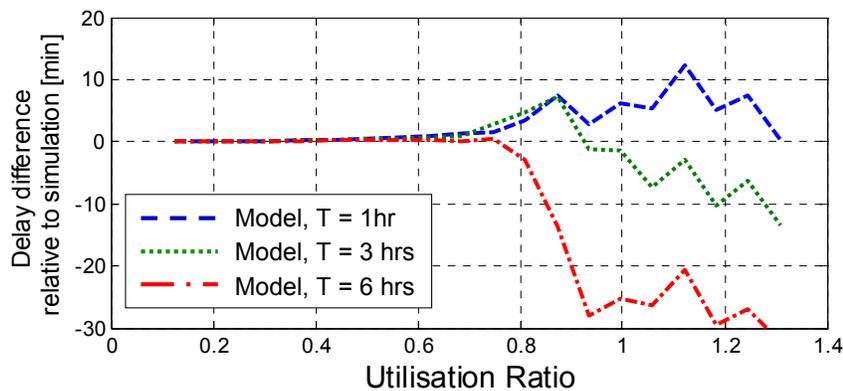
larger range of demand, and therefore a loss of accuracy. The model was thus run with three different bin sizes – 1 hr, 3 hrs and 6 hrs – and the results compared.

The model results and performance were each compared to the results and performance of a queuing simulation, averaged over 50 simulation runs, which provides a more accurate estimate of system delay. An  $M(t)/E_9(t)/1$  queuing system (exponentially distributed inter-arrival times / service times defined by an Erlang-k distribution with  $k = 9$  / single server) was simulated, as suggested by Malone (1995) for airport systems. The absolute and percentage difference between the average delay predicted by the model and by the simulation were calculated for each scenario. The demand run by the queuing simulation was defined per hour, as in Figure 7.

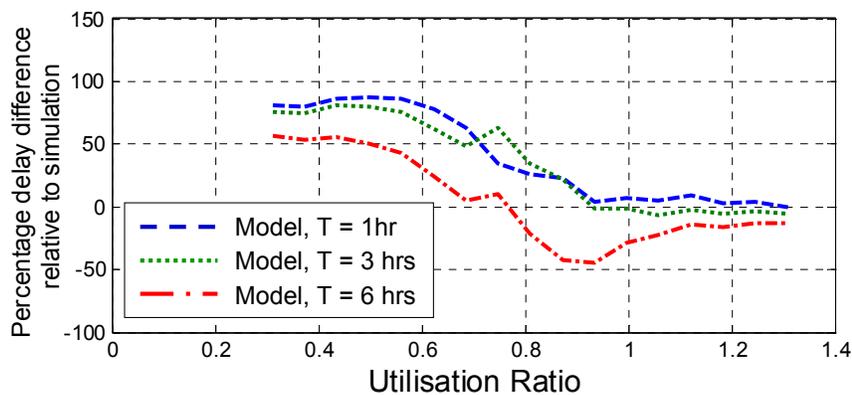
The model and simulation results were found to vary significantly by utilisation ratio, as expected, but not significantly by airport capacity or by schedule shape. The model results were found to vary significantly by bin size. Only results for a single airport capacity scenario (100 aircraft/hr) and schedule shape (banked) are thus presented, over a range of utilisation ratios and bin sizes. Differences between these results and those with other airport capacities and schedules are discussed. Average delay estimated by the simulation and by the model, for the three alternative bin sizes, is presented in Figure 8 over the range of utilisation ratios. Error bars are included for the simulation results, representing one standard deviation of the average delay calculated over the 50 simulation runs. The absolute differences between the simulation result and model results are presented in Figure 9. Percentage differences between the simulation result and model results are presented in Figure 10. Only percentage difference for utilisation ratios above 0.3 are presented in Figure 10 because the average delays at lower utilisations are so low that any difference results in a very large percentage difference, which although present, is not relevant. The average model and simulation runtimes are presented in the caption of Figure 8.



**Figure 8.** Average delay estimated by the simulation and model: banked schedule, airport capacity = 100 aircraft/hr. Simulation runtime = 2.4 sec; average model runtime =  $12 \times 10^{-6}$  sec.



**Figure 9.** Absolute delay difference between the simulation and model: banked schedule, airport capacity = 100 aircraft/hr.



**Figure 10.** Percentage delay difference between the simulation and model: banked schedule, airport capacity = 100 aircraft/hr.

In all the scenarios analysed both the simulation and the model predict a transition from low delays at low utilisation ratio, to approximately linearly increasing delay with utilisation ratio at high utilisation ratio. This is expected because, with increasing utilisation ratio and constant capacity, the scheduled demand shifts from predominantly well below capacity, for which delays are due to the stochastic nature of the system only, to predominantly near or above capacity, when the cumulative effects of delay building up during the day dominate.

The transition from low delay to linearly increasing delay occurs at a utilisation ratio of approximately 0.80 in all scenarios, which is the utilisation ratio at which the scheduled demand in a number of time bins reaches and exceeds capacity, even though the average demand from 6:00 to 24:00 remains below capacity. The transition between low delay and linearly increasing delay occurs at a slightly lower utilisation ratio for the banked schedule than the flat schedule. This is because of the presence of banks, which increase the effect of demand exceeding capacity in some time bins (the banks), when average demand is well below capacity. Average delay, at utilisation ratios at and beyond the transition between low delay to linearly increasing delay, are correspondingly slightly higher for the banked schedule than for the flat schedule.

The model over-predicts the simulation by between 50% and 90% at low utilisation ratios (Figure 10), although this corresponds to very low (less than 1 minute) absolute delay differences (Figure 9). The over-prediction is reduced as the bin size applied in the model is increased. This is because as the bin-size is increased, the range of utilisation ratios over which the steady state model can be applied, which is more accurate than the linear interpolation transition model, increases.

At high utilisation ratio – above 0.8 – the difference between the model and simulation results varies significantly with bin size. With a 1 hr bin size the model over-predicts the simulation by up to 30% (at a utilisation ratio of 0.8), or an absolute delay difference of up to 12 minutes (at a utilisation ratio of 1.3). With a bin size of 3 hrs the model initially over-predicts the simulation by up to 40%, and then transitions to under-predict the simulation by an absolute delay difference of up to 12 minutes. The model results fall within one standard deviation of the simulation results for both bin sizes of 1 hr and 3 hrs, up to a utilisation ratio of 1.25. With a bin size of 6 hrs the model under-predicts the simulation by up to 20%, or an absolute delay difference of up to 30 minutes, and does not fall within one standard deviation of the simulation results. As capacity increases, the variability in the delay difference between the model and simulation at high utilisation ratio typically reduces, with the model most closely correlating to the simulation in the highest capacity scenarios.

High utilisation ratios – above 1 – result in unrealistically high average delays in all scenarios – greater than two hours (Figure 8). These utilisation ratios are unlikely to occur in reality as airlines and passengers would respond to adjust the way in which they operate before these delays occurred. The utilisation ratios of most interest are instead between 0.8 and 1, which is the range of utilisation ratios within which most capacity constrained airports will most likely converge given airline and passenger responses to delay. At these utilisation ratios the model over-predicts the simulation by up to 30%, or an absolute delay difference of 8 minutes, for bin sizes of 1 hr and 3 hrs. For a bin size of 6 hrs, however, the model under-predicts the simulation by up to nearly 50%, or an absolute delay difference of nearly 30 minutes.

The model most closely approximates the simulation, over the full range of utilisation ratios and over the range likely to be encountered at airports (up to a 1), with a bin size of 3 hrs. With this bin size applied the model also falls within one standard deviation of the simulation results up to a utilisation ratio of 1.25. For this bin size, the maximum average delay difference relative to the simulation within the range of utilisation ratios likely to be encountered at airports does not exceed 8 minutes. The model's application is thus limited to where such an error in average delay is acceptable.

The average runtime is significantly lower for the model than for the simulation (in the order  $10^5$  times faster) in all scenarios run. The model run times remain approximately constant for all capacity scenarios, while the simulation run time increases as the capacity increases (it is a direct function of the number of aircraft processed).

## VII. Conclusions

The model developed produces results that fall within one standard deviation of the results of a queuing simulation – an accurate (but slow) model of airport system delays – with a maximum difference in average delay predicted by the model and simulation less than 8 min. The model's application is thus limited to where such an error in average delay is acceptable. These model results require specification of demand and capacity profiles in 3 hr time bins, and apply within the range of utilisation ratios likely to be encountered at airports.

The model run time is consistently  $10^5$  times faster than the simulation. This suggests that the model is useful if fast processing time is required, but only if average delay accuracy to within 8 minutes is acceptable.

The Aviation Integrated Modelling project (AIM) demands computational efficiency of a delay model that would allow it to be integrated within iteration loops including multiple other models, and solve on a personal computer for a number of airports globally. Also, because of the high uncertainties in inputs (e.g. GDP forecasts) and inherent in some of the other models to the AIM framework (e.g. demand, technology development, fleet turnover, global climate, and air quality models), and because the delay analysis is applied to highly congested airports, where average delay estimates in the future are likely to be high, average delay accuracy within 8 minutes is considered acceptable. The model developed is therefore thought to be useful for applications such as AIM.

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